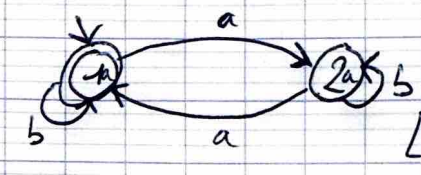
Théorie des langages
Correction du TD n°3

Exercice 1

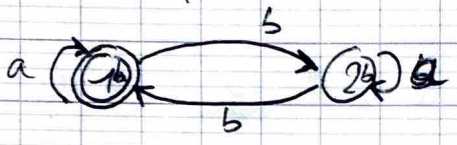
Automate qui reconnait un nombre pair de a.
On le note L_a :



L_a

$Q_a = \{1_a, 2_a\}$

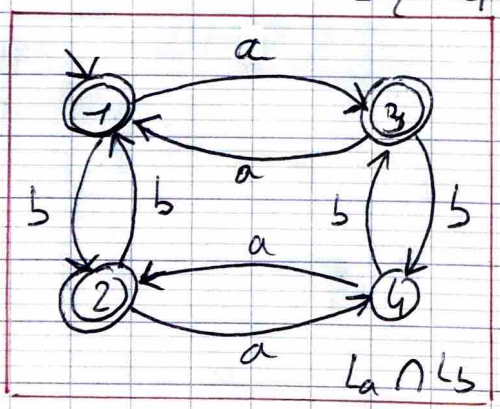
Automate qui reconnait des mots $|w|_b$ pair



$Q_b = \{1_b, 2_b\}$

Construire $L_a \cap L_b$

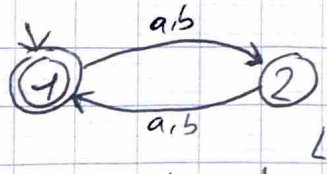
$Q_{a \cap b} = Q_a \times Q_b = \{(1_a 1_b), (1_a 2_b), (2_a 1_b), (2_a 2_b)\}$
$= \{1, 2, 3, 4\}$



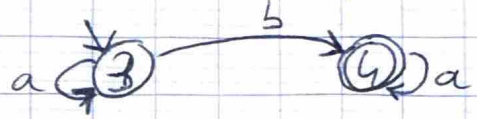
Les liens sont effectués par accessibilité dans chaque automates.

Exercice 2

- L_a reconnaît les mots pairs
- L_b reconnaît les mots w tel que $|w|_b \leq 1$

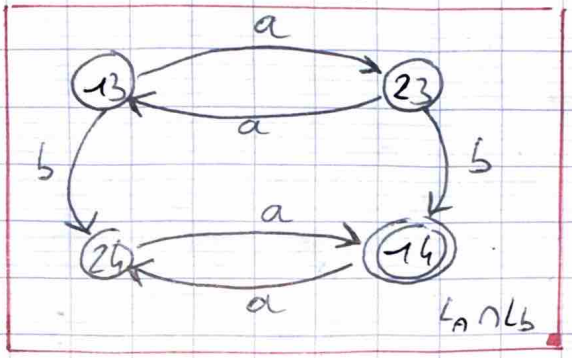


L_a



Automate $L_a \cap L_b$.

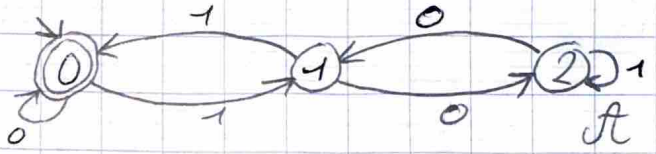
$Q_a = \{1, 2\}$
$Q_b = \{3, 4\}$
$\Big\}$ $Q_a \times Q_b = \{(1,3), (14), (23), (24)\}$.



On privilégie le fait de ne pas croiser les arrêtes.

Exercice 3 $\Sigma = \{0, 1\}$

On considère l'automate suivant :



Question (a).

$L(\mathcal{A}) = L_0$

$$\begin{cases} L_0 = 0L_0 + 1L_1 + \varepsilon \\ L_1 = 1L_0 + 0L_2 \\ L_2 = 1L_2 + 0L_1 \end{cases}$$

$\xrightarrow{a}$ Règle d'Arden

$L_2 = 1L_2 + 0L_1 = 1^*0L_1$ $\xrightarrow{A}$

$L_1 = 1L_0 + 01^*0L_1 = (01^*0)^*1L_0$ $\xrightarrow{A}$

$L_0 = 0L_0 + 1(01^*0)^*1L_0 + \varepsilon$

$= (0 + 1(01^*0)^*1)L_0 + \varepsilon$ $\xrightarrow{a}$ $= (0 + 1(01^*0)^*1)^*\varepsilon$

Question (b)

les multiples de 3

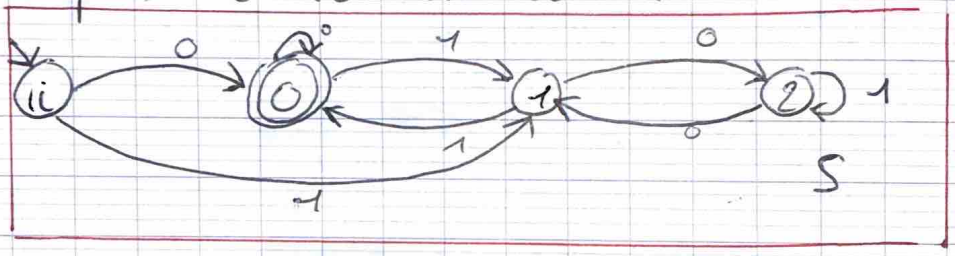
- $0 = (0)_2$
- $3 = (11)_2$
- $6 = (110)_2$
- $9 = (1001)_2$
- ⋮

Question (c)

Automate standard

↳ 1 seul état initial

↳ pas de retour sur lui



Question (d)

$L(S) = L_i$

$$\begin{cases} L_i = 0L_0 + 1L_1 \\ L_0 = 1L_1 + \varepsilon + 0L_0 \\ L_1 = 0L_2 + 1L_0 \\ L_2 = 1L_2 + 0L_1 \end{cases}$$

$L_2 = 1L_2 + 0L_1 \xrightarrow{A} = 1^*0L_1$

$L_1 = 01^*0L_1 + 1L_0 \xrightarrow{A}$

$= (01^*0)^*1$

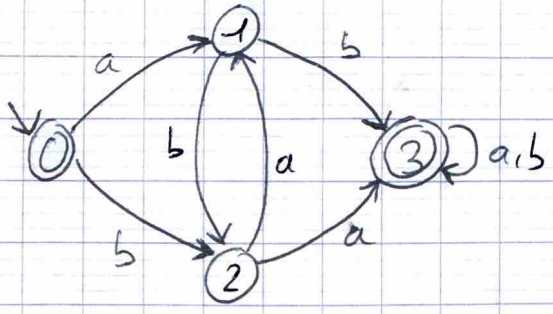
$L_0 = 1(01^*0)^*1 + 0L_0 + \varepsilon$

$= 0^*1(01^*0)^*1 + \varepsilon$

Exercice 5

l'exercice 4 permet d'évaluer les mêmes compétences.

Automate $\mathcal{A}_1$

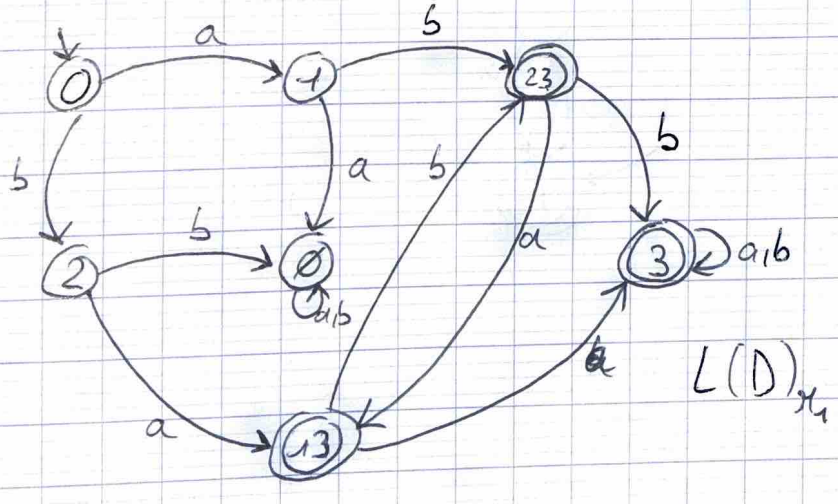


I	✓						
F					✓	✓	✓
	0	1	2	∅	23	13	3
a	1	∅	13	∅	13	3	3
b	2	23	∅	∅	3	23	3

Initial

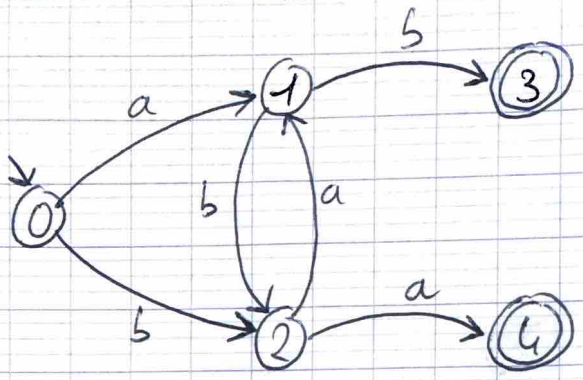
final

On obtient



$L(D)_{\mathcal{A}_1}$

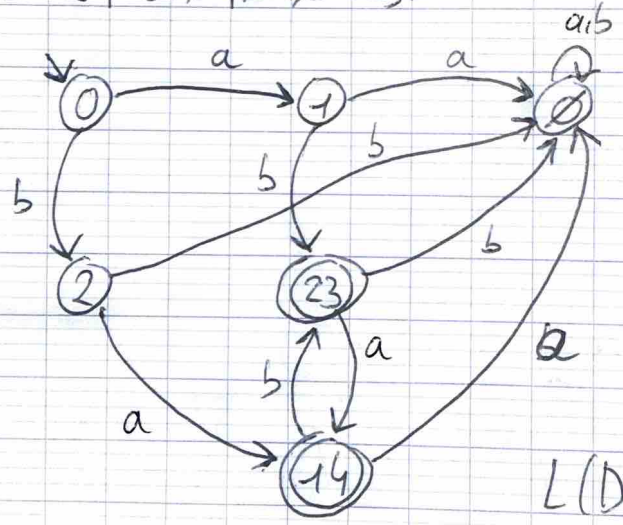
Automate $\mathcal{A}_2$



I	∂					
F				∂		∂
	0	1	2	23	∅	14
a	1	∅	14	14	∅	∅
b	2	23	∅	∅	∅	23

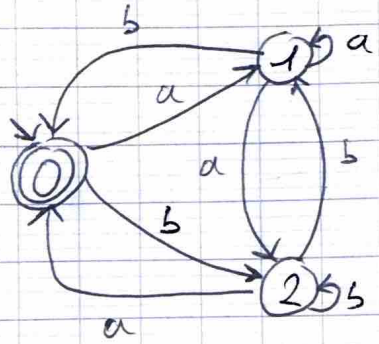
Initial

Final



$L(D)_{\mathcal{A}_2}$

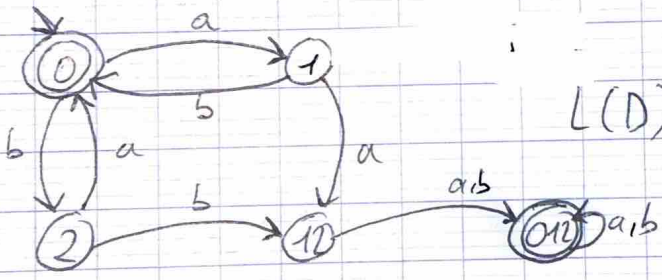
Automate $\mathcal{A}_3$



I	∂				
F	∂			·	∂
	0	1	2	12	012
a	1	12	0	012	012
b	2	0	12	012	012

Initial

Final



$L(D)_{\mathcal{A}_3}$.